# COLLEGE REQUIREMENTS

IN

# ALGEBRA

*A FINAL REVIEW*

BY

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# PREFACE.

THE Williston students have found these reviews so serviceable that a more convenient form has become necessary. From a wide collection of college papers about four hundred examples, illustrating nearly every principle in Algebra, were selected and carefully arranged by subjects. Whenever a suitable one could not be so obtained, an original problem or one from foreign texts was inserted. The parallel sections are for the use of two divisions and for recitation-room drill.

Colleges will find the work useful as an initial review; while college candidates may be assured of entering if they perform all the examples without aid.

Suggestions in regard to the work will be gladly received.

G. P. T.

WILLISTON SEMINARY,
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# CONTENTS.

## SECTION I.

## SIGHT PROBLEMS.

1. Of 452 students who tried the examinations last June, $x$ were men, the rest were women. In all, $z$ students failed. How many of the failures were men?

2. The sum of 3 consecutive numbers exceeds the middle number by 10. What are the numbers?

3. In how many weeks will $x$ horses eat 50 bushels of oats, if one horse eats $y$ bushels in a week?

4. A is 20 years old, and B is $-2$ years older; what is the age of B? [*Harvard.*]

5. Two men working separately can do a piece of work in $x$ days and $y$ days respectively; find an expression for the time in which both can do it working together. [*Harvard.*]

6. A pole 100 feet high, standing on the side of a hill, breaks off so as to form a right angle at the break, with the top resting on the hill 75 feet from the foot of the pole; where did the pole break? [*Cornell.*]

7. Two steamers ply between the same two ports a distance of 420 miles. One travels half a mile an hour faster than the other, and is two hours less on the journey; find the rates. [*Vassar.*]

## SECTION II.

## PARENTHESES AND EVALUATION.

1. $3x-(5x-[4x-(y-x)])-(-x-3y)$.

2. $a-(2b+[3c-3a-(a+b)]+2a-(b+3c))$.

3. $ax+b(x+c)+c^2-[(a-b)x-(b-c)(b+c)]$.

4. $ab-c(x-b)-[(x+c)(x-c)-c(b-\{c-x\})-x^2]$.

5. $-[13x-14y-3\{2x-2(-y+2x)-7y\}]$.

6. $a+b\{x-3a(y-2x)+3x[4a+2(4b+3)]\}$.

7. $\frac{1}{4}\left\{x-5(y-x)\right\}-\frac{3}{2}\left\{\frac{1}{3}\left(y-\frac{x}{3}\right)-\frac{2}{9}\left[a-\frac{3}{4}\left(y-\frac{4x}{5}\right)\right]\right\}$.

8. Evaluate $a+2x-\{b+y-[a-x-(b-2y)]\}$.

9. Evaluate $x-(\sqrt{x+1}+2)-\frac{x-\sqrt[3]{x}}{x-4}$ when $x=8$.

10. Substitute $x+3$ for $y$ in $y^3+2y^2-15y-36$.

## SECTION III.

## PARENTHESES AND EVALUATION.

1. $2a-[5b+\{3c-(a+[2b-3a+4c])\}]$.

2. $3a-\{3a-[3a-(3a-\overline{3a-3a})-3a]-3a\}-3a$.

3. $(a+b)x-(b-c)c-[(b-x)b-(b-c)(a+c)]-ax$.

4. $3x-4y+5[-4x-\{3y-(2+7x-2y)-4\}]$.

5. $3c^2+c(2a-[5c-\{3a+\overline{c-4a}\}])$.

6. $x^3y^2-\left(-xy^2+x^3-\frac{x^4}{y}\right)xy-x^2[-\{y^3-y(xy-x^2)\}]$.

7. $(a^2-b^2)c-(a-b)(a[b+c]-b[a-c])$.

8. Evaluate $\sqrt[3]{a}+\sqrt[3]{ab}+\sqrt[3]{b}$ when $a=8,\ b=64$.

9. Evaluate $\sqrt[3]{a}\left\{b^3+\sqrt[4]{\frac{a^2+2a\overline{b}+\overline{b^2}}{b^2}}\right\}$ when $a=8,\ b=1$.

10. Substitute $y-3$ for $x$ in $x^3+2x^2-15x-36$.

## SECTION IV.

## FACTORING.

1. Difference of Squares: $(x^2+y^2-z^2)^2-4x^2y^2.$

2. Cubes: $x^6-1.$

3. Simple Trinomial: $x^4-13x^2y^2+36y^4.$

4. Complex Trinomial: $6x^2-5xy-6y^2.$

5. Literal Trinomial: $x^2+\left(a+\frac{1}{a}\right)xy+y^2.$

6. Four Terms: $a^4+a^2b^2-b^2c^2-c^4.$

7. Five Terms: $6a^2+5ab-6b^2-6ac+4bc.$

8. Six Terms: $2a^2-ab-ac-6b^2-5bc-c^2.$

9. Perfect Square: $16x^6+16x^7-4x^8-4x^9+x^{10}$

10. Imperfect Square: $x^4-7x^2y^2+y^4.$

11. Radicals: $x^4+1.$

12. Separation: $x^3-6x^2+11x-6.$

13. Literal Exponents: $x^{2m}+\frac{1}{2}x^m+\frac{1}{16}.$

14. Parentheses: $(1+y)^2-2x^2(1-y^2)+x^4(1-y)^2.$

## SECTION V.

## FACTORING.

1. Difference of Squares: $1-(x^2+y^2)+2xy.$

2. Cubes: $a^{12}-b^{12}.$

3. Simple Trinomial: $x^2-2x-3.$

4. Complex Trinomial: $8a^2+2a-3.$

5. Literal Trinomial: $1-(m^2+n^2)x^2+m^2n^2x^4.$

6. Four Terms: $m^3x+m^3y-n^3x-n^3y.$

7. Five Terms: $ax^5-3ax^4+2ax^3+ax^2-ax.$

8. Six Terms: $6a^2-5ab+14ac+b^2-5bc+4c^2.$

9. Perfect Square: $67x^2+49+9x^4-70x-30x^3.$

10. Imperfect Square: $9a^4-40a^2b^2+16b^4.$

11. Radicals: $a+b.$

12. Separation: $x^3+10x^2+29x+20.$

13. Literal Exponents: $x^n-y^n$

14. Parentheses: $a(a-1)x^2-(a-b-1)xy-b(b+1)y^2.$

## SECTION VI.

### H. C. F., L. C. M., EVOLUTION.

Express both H. C. F. and L. C. M. in factors: —

1. $6x^2+13x-5$,
   $3x^3+2x^2+2x-1$. [*Princeton.*]

2. $b^4-6b^2-8b-3$,
   $4b^3-12b-8$. [*Yale.*]

3. $6x^4-13x^3+3x^2+2x$,
   $6x^4-9x^3+15x^2-27x-9$. [*Brown.*]

4. $4x^4-9x^2+6x-1$,
   $6x^3-7x^2+1$. [*Johns Hopkins.*]

5. $6x^4-5x^3-10x^2+3x-10$,
   $4x^3-4x^2-9x+5$. [*Harvard.*]

6. $8x^3+27$,
   $16x^4+36x^2+81$,
   $6x^2+5x-6$. [*Vassar.*]

7. Square Root of $4a^4+12a^3+5a^2-6a+1$.

8. Cube Root of $8a^6+12a^5-6a^4-11a^3+3a^2+3a-1$.

## SECTION VII.

### H. C. F., L. C. M., EVOLUTION.

Express both H. C. F. and L. C. M. in factors: —

1. $2x^5 - 11x^2 - 9$,
   $4x^5 + 11x^4 + 81$. [*Johns Hopkins.*]

2. $8x^2 + 2x - 3$,
   $12x^3 + 10x^2 - 4$. [*Amherst.*]

3. $15a^2x^3 - 20a^2x^2 - 65a^2x - 30a^2$,
   $12bx^3 + 20bx^2 - 16bx - 16b$. [*Brown.*]

4. $x^3 - 3x - 2$,
   $x^5 - 2x^4 - x + 2$. [*Dartmouth.*]

5. $x^2 - 1$,
   $x^2 - 2x - 3$,
   $6x^2 - x - 20$. [*Technology.*]

6. $2x^4 + x^3 - 8x^2 - x + 6$,
   $4x^4 + 12x^3 - x^2 - 27x - 18$,
   $4x^4 + 4x^3 - 17x^2 - 9x + 18$. [*Harvard.*]

7. Square Root of $8x^6 + \frac{1}{8x^6} - 8x^4 + 2x^2 - \frac{1}{x^2} + 2$.
   [*Bryn Mawr.*]

8. Cube Root of $x^6 - 6x^5 + 3x^4 + 28x^3 - 9x^2 - 54x - 27$.

## SECTION VIII.

## FRACTIONS.

1. Reduction :

$$\frac{4x^2+3x-10}{4x^3+7x^2-3x-15}.$$

2. Addition and Subtraction :

$$\frac{a^2-bc}{(a-b)(a-c)}+\frac{b^2+ca}{(b+c)(b-a)}+\frac{c^2+ab}{(c-a)(c+b)}.$$

3. Multiplication : $\left[\frac{x+y}{x-y}-\frac{x-y}{x+y}-\frac{4y^2}{x^2-y^2}\right]$ by $\left[\frac{x+y}{2y}\right]$.

4. Division : $\left[\frac{x^2+y^2}{x^2-y^2}-\frac{x^2-y^2}{x^2+y^2}\right]$ by $\left[\frac{x-y}{x+y}+\frac{x+y}{x-y}\right]$.

5. Complex :

$$\cfrac{1}{12+\cfrac{1}{1+\cfrac{1}{2+\frac{1}{3}}}}.$$

6. Evaluation :

If $x=\frac{4ab}{a+b}$, find the value of $\frac{x+2a}{x-2a}+\frac{x+2b}{x-2b}$.

7. Miscellaneous :

$$\frac{\frac{1-x^2}{1+y}\left(\frac{x}{1+x}-1\right)}{1-\left(\frac{1}{1-y}-\frac{x^2+y^2-x+y}{1-y^2}\right)}.$$

## SECTION IX.

## FRACTIONS.

**1.** Reduction :

$$\frac{\dfrac{2x}{3}}{\dfrac{1}{x}-\dfrac{2x^3+11x^2-43x-24}{14x^3-31x^2-31x-6}}.$$

**2.** Addition and Subtraction :

$$\frac{1}{x(x-a)(x-b)}+\frac{1}{a(a-x)(a-b)}+\frac{1}{b(b-x)(b-a)}.$$

**3.** Multiplication :

$$\frac{b-y}{a^3+y^3}\times\frac{ca+cy}{b^2+by}\times\frac{b^6+y^6}{b^2+y^2}\times\frac{b}{c}.$$

**4.** Division :

$$\left[\frac{1-x^3}{1+x^3}-\frac{1-x}{1+x}\right] \text{ by } \left[\frac{1+x^2}{1-x^2}+\frac{1+x}{1-x}\right].$$

**5.** Complex :

$$\frac{1}{x+\dfrac{2}{1-\dfrac{x-2}{2x+1}}}.$$

**6.** Evaluation :

If $\dfrac{x}{y+z}=a,\ \dfrac{y}{x+z}=b,\ \dfrac{z}{x+y}=c,$

find value of $\dfrac{a}{1+a}+\dfrac{b}{1+b}+\dfrac{c}{1+c}$.

**7.** Miscellaneous :

$$\frac{\left(a^2+\dfrac{b^4}{a^2+b^2}\right)\left(a^2-b^2\right)}{1-\dfrac{b}{a+b}-\dfrac{1}{2}\left(1-\dfrac{a-b}{a+b}\right)}.$$

## SECTION X.

## SIMPLE EQUATIONS.

1. $\frac{7x+9}{4}-\left(x-\frac{2x-1}{9}\right)=7.$

2. $\frac{5x+2}{3}-\left(3-\frac{3x-1}{2}\right)=\frac{3x+19}{2}-\left(\frac{x+1}{6}+3\right).$

3. $\frac{8x+23}{20}-\frac{5x+2}{3x+4}=\frac{2x+3}{5}-1.$

4. $\frac{3(y-1)}{16}-\frac{5}{12}(y-4)=\frac{2}{5}(y-6)+\frac{5}{48}.$

5. $12\{3x-.25(x-4)-.3(5x+14)\}=47.$

6. $\frac{x}{2}-\frac{a-bcx}{2bc}=\frac{x}{6c}-\frac{ac-4bx}{3bc}.$

7. $\frac{x}{2a}-3+\frac{x}{4a^3}=\frac{x}{3a^2}-2a(2-3a).$

8. At what time between 7 and 8 o'clock are the hands of a watch (*a*) together?
   (*b*) opposite?
   (*c*) at right angles?

## SECTION XI.

## SIMPLE EQUATIONS.

1. $2x - \left(x - \frac{x-1}{3}\right) = \frac{5x}{4}$.

2. $3z - \frac{z-4}{4} - 4 = \frac{5z+14}{3}$.

3. $\frac{x+10}{3} - \frac{2}{5}(3x-4) + \frac{(3x-2)(2x-3)}{6} = x^2 - \frac{2}{15}$.

4. $\frac{1}{2}\left(x - \frac{a}{3}\right) - \frac{1}{3}\left(x - \frac{a}{4}\right) + \frac{1}{4}\left(x - \frac{a}{5}\right) = 0$.

5. $3.3x - \frac{.72x - .55}{.5} = .1x + 9.9$.

6. $\frac{ax-b}{b} - \frac{1-x}{a} + m = 0$.

7. $3ax - 2bx - \frac{1}{3}c - \frac{1}{4}mx = \frac{2}{3}c + \frac{3}{4}mx - n - bx + 2ax$.

8. A man bought a horse, and expected to sell it at 10 % profit; but had to sell it for \$50 less than he expected, and then found he had lost 15 % on what it cost him. What did he pay for the horse?

## SECTION XII.

## SIMULTANEOUS EQUATIONS.

1. Simple:
$$\left.\begin{aligned} ax + by &= c \\ a'x + b'y &= c' \end{aligned}\right\}$$

2. Reciprocals:
$$\left.\begin{aligned} \frac{1}{3x} + \frac{1}{5y} &= \frac{2}{9} \\ \frac{1}{5x} + \frac{1}{3y} &= \frac{1}{4} \end{aligned}\right\}$$

3. Parentheses:
$$\left.\begin{aligned} (a+b)x - (a-b)y &= 4ab \\ (a-b)x - (a+b)y &= 0 \end{aligned}\right\}$$

4. Three Quantities:
$$\left.\begin{aligned} ax + by &= l \\ cy + dz &= m \\ ex + fz &= n \end{aligned}\right\}$$

5. Fractions:
$$\left.\begin{aligned} \frac{m}{x} + \frac{n}{y} &= a \\ \frac{n}{x} + \frac{m}{y} &= b \end{aligned}\right\}$$

6. Numerical:
$$\left.\begin{aligned} x - 2y + 3z &= 2 \\ 2x - 3y + z &= 1 \\ 3x - y + 2z &= 9 \end{aligned}\right\}$$

7. *Problem.* A firm has Java coffee at $a$ cents per pound and Mocha at $b$ cents per pound. How much of each is there in a mixture of $a - b$ pounds if it can be sold at $c$ cents a pound without loss?

## SECTION XIII.

## SIMULTANEOUS EQUATIONS.

1. Simple:
$$\left.\begin{aligned} x+y &= \frac{2(a^2+b^2)}{a^2-b^2} \\ x-y &= \frac{4ab}{a^2-b^2} \end{aligned}\right\}$$

2. Reciprocals:
$$\left.\begin{aligned} \frac{7}{\sqrt{x}}+\frac{4}{\sqrt{y}} &= 4 \\ \frac{1}{\sqrt{x}}+\frac{2}{\sqrt{y}} &= 1 \end{aligned}\right\}$$

3. Parentheses:
$$\left.\begin{aligned} 7(x+y)+3(x-y) &= 80 \\ 7(x+y)-3(x-y) &= 32 \end{aligned}\right\}$$

4. Three Quantities:
$$\left.\begin{aligned} y+z &= a \\ x+z &= b \\ x+y &= c \end{aligned}\right\}$$

5. Fractions:
$$\left.\begin{aligned} &\frac{x}{a}+\frac{y}{b}, \\ &=\frac{x}{a}+\frac{z}{c} \\ &=\frac{y}{b}+\frac{z}{c}=1 \end{aligned}\right\}$$

6. Numerical:
$$\left.\begin{aligned} 2x+\ 4y+27z &= 28 \\ 7x-\ 3y &= 15z=\ 3 \\ 9x-10y &= 33z=\ 4 \end{aligned}\right\}$$

7. *Problem.* A and B walk in a circle whose circumference is $C$. If they start from the same point and go in opposite directions they meet in 4 hours. If they start from opposite points and walk around the circle in the same direction they meet in 8 hours. Find the rate at which each one walks.

## SECTION XIV.

## THEORY OF EXPONENTS.

1. Discussion: $\frac{a^0}{b^m}$; $x^{\frac{3}{4}}$.

2. Proof: $(a^m)^n = a^{nm}$.

3. Removal of negative exponents: $\frac{1 - a^{-2} - y^2}{1 - x^{-3}y^{-2} + x^{-2}}$.

4. Combination: $\left(\frac{a^{\frac{1}{2}}b}{c^{-\frac{3}{4}}}\right)^{\frac{1}{3}} \times \left(\frac{\sqrt{c}}{e^{\frac{4}{3}}}\right)^{-\frac{1}{2}} \div \left(\frac{2}{b}\right)^{\frac{3}{3}}$.

5. Multiplication: $(a^{\frac{5}{2}} - a^2b^{\frac{1}{3}} + a^{\frac{3}{2}}b^{\frac{2}{3}} - ab + a^{\frac{1}{2}}b^{\frac{4}{3}} - b^{\frac{5}{3}}$ by $(a^{\frac{1}{2}} + b^{\frac{1}{3}})$.

6. Division: $x^2y^{-\frac{4}{3}} - 2 + x^{-2}y^{\frac{4}{3}}$ by $x^{\frac{1}{2}}y^{-\frac{1}{3}} - x^{-\frac{1}{2}}y^{\frac{1}{3}}$.

7. Involution: $\left[\sqrt[26]{\sqrt[3]{x^2} - \sqrt{\left(\frac{\sqrt{x}}{\sqrt[3]{x}}\right)^5}}\right]^3$.

8. Evolution:

$$1 + 4y^{-\frac{1}{3}} - 2y^{-\frac{2}{3}} - 4y^{-1} + 25y^{-\frac{4}{3}} - 24y^{-\frac{5}{3}} + 16y^{-2}.$$

9. Reduction: $\frac{x^{\frac{2}{3}} - 4\sqrt[3]{x^{-2}}}{\sqrt[3]{x^2} + 4 + 4x^{-\frac{2}{3}}}$.

10. *Problem.* Find the number whose cube root is one-fifth of its square root.

## SECTION XV.

## THEORY OF EXPONENTS.

1. Discussion : $\left[\frac{(a^{m-n})^{m+n}(a^n)^{n+p}}{(a^m)^{m-n}}\right]^{\frac{1}{n}}$.

2. Proof : $a^0 = 1, \quad a^{-n} = \frac{1}{a^n}$.

3. Removal of negative exponents : $\frac{a^{-2} + b^{-5}}{c^{-4} - d^{-7}}$.

4. Combination : $a^{\frac{1}{2}}y^{\frac{1}{2}} \times \left(\frac{y^{\frac{1}{4}}}{x^{\frac{1}{6}}}\right)^2 \div \frac{y^{-\frac{1}{4}}}{x^{\frac{1}{4}}}$.

5. Multiplication :

$$(a^{\frac{2}{3}} - a^{\frac{1}{3}} + 1 - a^{-\frac{1}{3}} + a^{-\frac{2}{3}}) \text{ by } (a^{\frac{1}{3}} + 1 + a^{-\frac{1}{3}}).$$

6. Division : $(a^{x-3})^x \div \left(\frac{\sqrt[8]{b}}{a}\right)^{x+1}$.

7. Involution : $\left[\frac{a^{\frac{x}{x-y}}\sqrt[y]{b^{-(y-x)}}}{a^{\frac{y}{x-y}}\sqrt[y]{a^{x+y}b^x}}\right]^{-y}$.

8. Evolution : $x^3 + 2x^{\frac{5}{2}} - 3x^2 - 4x^{\frac{3}{2}} + 4x$.

9. Reduction : $\frac{x^{\frac{4}{3}} - 8x^{\frac{1}{3}}y}{x^{\frac{2}{3}} + 2\sqrt[3]{xy} + 4y^{\frac{2}{3}}}$.

10. Equation : $\sqrt{x^{\frac{3}{5}}} = 2\sqrt{2}$; find $x$.

## SECTION XVI.

## RADICALS.

1. Reduction : $\dfrac{x+1}{x-1}\sqrt{\dfrac{x-1}{x+1}}$.

2. Addition and Subtraction :

$$\sqrt{9ab^3} - \sqrt{a^3b} + \sqrt{a^3b - 6a^2b^2 + 9ab^3}.$$

3. Arrangement : $\sqrt[6]{2^5}$, $\sqrt{3}$, $\sqrt[3]{5\frac{1}{5}}$.

4. Multiplication : $[x - \frac{1}{2}(1 - \sqrt{-3})][x - \frac{1}{2}(1 + \sqrt{-3})]$.

5. Division : $\dfrac{(\sqrt[5]{a}\,\sqrt[4]{b})^3}{\sqrt[3]{c^2}}$ by $\dfrac{\sqrt[6]{c^2b^5}}{\sqrt[5]{a^7}}$.

6. Involution and Evolution : $[2\sqrt{3} + 3\sqrt{2} + \sqrt{6}]^2$.

7. Rationalization : $\dfrac{\sqrt{x} - \sqrt{x+y}}{\sqrt{x} + \sqrt{x+y}}$.

8. Imaginaries : $\sqrt{-9x^4} + \sqrt{-16x^4} - \sqrt{-(x-1)^2x^2}$.

9. Binomial Surds : $\sqrt{41 + 12\sqrt{5}}$.

10. Radical Equation : $\sqrt{x+2} - \sqrt{x-2} = 2x$.

## SECTION XVII.

## RADICALS.

1. Reduction : $\sqrt[3]{(a^3-1)(a-1)^2}$.

2. Addition and Subtraction :

$$2\sqrt{125}-\sqrt[4]{\tfrac{25}{16}}+\sqrt[3]{81}-(-512)^{\frac{1}{3}}+\sqrt[3]{192}.$$

3. Arrangement : $\sqrt{3},\ \sqrt[3]{6},\ \sqrt[4]{10}$.

4. Multiplication : $\left(x-\frac{1-\sqrt{3}}{2\sqrt{2}}\right)\left(x-\frac{1+\sqrt{3}}{2\sqrt{2}}\right)\left(x+\frac{1}{\sqrt{2}}\right)$.

5. Division : $\sqrt{2\sqrt[3]{2}}\div\sqrt{\sqrt{2\sqrt[3]{2}}}$.

6. Involution and Evolution : $\left(\sqrt{\sqrt{\frac{x\sqrt{y}}{\sqrt[3]{xy}}}}\right)^3$.

7. Rationalization : $\frac{\sqrt{3}}{2-\sqrt{3}}-\frac{2-\sqrt{-2}}{2+\sqrt{-2}}$.

8. Imaginaries : $[2\sqrt{-3}-5\sqrt{-2}]^2$.

9. Binomial Surds : $\sqrt{14+4\sqrt{6}}$.

10. Radical Equations : $\sqrt{x+5}+\sqrt{x-8}=\sqrt{3}$.

## SECTION XVIII.

## QUADRATICS.

1. Simple: $12x^2 + x - 1 = 0.$

2. Literal: $a(x^2 - x) + b(x^2 + x) = \frac{ab}{a+b}.$

3. Fractional: $\frac{(4a^2 - b^2)(x^2 + 1)}{4a^2 + b^2} = 2x.$

4. Parentheses: $(x-2)(x+3)(x^2 + 3x - 4) = 0.$

5. Rationalization: $\frac{3x - \sqrt{x^2 - 8}}{x - \sqrt{x^2 - 8}} = x + \sqrt{x^2 - 8}.$

6. Fractional Exponents: $x^3 - x^{\frac{3}{2}} = 256.$

7. Quadratic Form: $(ax - b)^2 - 4a(ax - b) = \frac{9}{4}a^2.$

8. Radicals: $\frac{10x}{\sqrt{10x - 9}} - \sqrt{10x + 2} = \frac{2}{\sqrt{10x - 9}}.$

9. Cubic Equation: $x^3 - x^2 - x + 1 = 0.$

10. Formation:
    Find equation with roots $(a+b-c)$ and $(a-b+c)$.

11. *Problem.* Tristram is ten years younger than Launcelot; and the product of the ages they attained in 1890 is 96. Find the ages they attain in 1908. [*Harvard.*]

## SECTION XIX.

## QUADRATICS.

1. Simple: $91x^2 - 2x - 45 = 0.$

2. Literal: $\dfrac{(a+2b)x}{a-2b} = \dfrac{a^2}{a-2b} - \dfrac{4b^2}{x}.$

3. Fractional: $\dfrac{x+1}{c} - \dfrac{2}{cx} = \dfrac{x+2}{ax-bx}.$

4. Parentheses: $(x-1)(x-2)(x^2-6x+9) = 0.$

5. Binomial Surds: $(7-4\sqrt{3})x^2 + (2-\sqrt{3})x = 2.$

6. Fractional Exponents: $4\sqrt[4]{x} + \sqrt{x} = 21.$

7. Quadratic Form: $x^2 - 2x + 6\sqrt{x^2-2x+5} = 11.$

8. Radicals: $\sqrt{x+a} + \sqrt{x} + \sqrt{x-a} = 0.$

9. Bi-quadratic: $x^4 - 2x^3 + x - 2 = 0.$

10. Formation:

    Find equation with the roots $\dfrac{(a+b)^2}{a-b}$ and $(b-a)$.

11. *Problem.* A cask $P$ is filled with 50 gallons of water, and a cask $Q$ with 40 gallons of brandy. $x$ gallons are drawn from each cask, mixed and replaced, and the same operation is repeated. After the second replacement there are $8\frac{7}{8}$ gallons of brandy in $P$. Find $x$.

## SECTION XX.

## SIMULTANEOUS QUADRATICS.

**1.** Substitute: $\left.\begin{aligned} 2x+3y &= 12 \\ 3x^2-2xy &= 15 \end{aligned}\right\}$

**2.** Obtain $xy$: $\left.\begin{aligned} x-y &= 4 \\ x^2+y^2 &= 106 \end{aligned}\right\}$

**3.** Let $y=vx$: $\left.\begin{aligned} x^2+10xy &= 11 \\ 5xy-3y^2 &= 2 \end{aligned}\right\}$

**4.** Divide: $\left.\begin{aligned} x^3-y^3 &= 56 \\ x^2+xy+y^2 &= 28 \end{aligned}\right\}$

**5.** Retain Fractions: $\left.\begin{aligned} \frac{1}{x}-\frac{1}{y} &= 0 \\ \frac{1}{x^2}+\frac{1}{y^2} &= 20 \end{aligned}\right\}$

**6.** Yale: $\left.\begin{aligned} x^2-y^2 &= 9 \\ \frac{15}{x}:\frac{21}{y} &:: 3:7 \end{aligned}\right\}$

**7.** Wellesley: $\left.\begin{aligned} x^2+xy &= 15 \\ xy-y^2 &= 2 \end{aligned}\right\}$

**8.** Michigan University: $\left.\begin{aligned} x+y-\sqrt{xy} &= 7 \\ x^2+y^2+xy &= 133 \end{aligned}\right\}$

**9.** Columbia; Mines: $\left.\begin{aligned} x^2+y+3x &= 73-2xy \\ y^2+x &= 44-3y \end{aligned}\right\}$

## SECTION XXI.

## SIMULTANEOUS QUADRATICS.

1. Substitute: $\left.\begin{aligned} x-3y&=1\\ x^2-2xy+9y^2&=17 \end{aligned}\right\}$

2. Obtain $xy$: $\left.\begin{aligned} x-y&=10\\ x^2+y^2&=58 \end{aligned}\right\}$

3. Let $y=vx$: $\left.\begin{aligned} x^2+3xy&=-8\\ y^2-xy&=12 \end{aligned}\right\}$

4. Divide: $\left.\begin{aligned} x^2-xy+y^2&=9\\ x^4+x^2y^2+y^4&=243 \end{aligned}\right\}$

5. Retain Fraction: $\left.\begin{aligned} \frac{1}{x}+\frac{1}{y}&=7\\ \frac{1}{x^2}-\frac{1}{y^2}&=25 \end{aligned}\right\}$

6. Vassar: $\left.\begin{aligned} x^3+y^3&=56\\ x+y&=2 \end{aligned}\right\}$

7. Princeton: $\left.\begin{aligned} x^2+xy+y^2&=52\\ xy-x^2&=8 \end{aligned}\right\}$

8. Univ. Penn.: $\left.\begin{aligned} x^2-xy+y^2&=7\\ x^4+x^2y^2+y^4&=133 \end{aligned}\right\}$

9. Harvard: $\left.\begin{aligned} \frac{1}{x+y}-\left(\frac{y}{a(x-y)}-\frac{x+6a}{x^2-y^2}\right)&=0\\ y:(7x-2y)&=(b-a):(2a-9b) \end{aligned}\right\}$

## SECTION XXII.

## INEQUALITIES, PROPORTION, VARIATION.

1. Prove that the square of half the sum of any two quantities $<$ half the sum of their squares.

2. Prove $a^3 > a + \frac{1}{a} - 1$ if $a > 1$.

3. What two numbers whose difference is $d$ are to each other as $a$ is to $b$?

4. If $x - y$ is a mean proportional between $y$ and $(y + z - 2x)$, show that $x$ is a mean proportional between $y$ and $z$.

5. Prove that if

$$2x : y :: a : b, \quad \tfrac{1}{2}a - x : \tfrac{1}{2}a + x :: b - y : b + y.$$

6. Prove that a proportion taken by inversion is a true proportion.

7. $A$ varies jointly as $B$ and $C$; and $A = 6$ when $B = 3$, $C = 2$. Find $A$ when $B = 5$, $C = 7$.

8. $A$ varies as the square of $B$, and inversely as the square of $C$, and $A = 4$ when $B = 1$, $C = 2$. What is the value of $\frac{B^2 + A^2}{B^2 - A^2}$ when $B = 2$, $C = 2$?

9. The volume of a sphere varies as the cube of the radius. If 3 spheres with radii 9, 12, and 15 inches are melted into a single sphere, find its radius.

## SECTION XXIII.

## INEQUALITIES, VARIATION, PROPORTION.

1. Prove that $\left(\frac{a}{2+b}\right)^2 < ab$.

2. Find the limit of $x$ in

$$(3x+2)(x-3) > (x+4)(3x-1) - 3.$$

3. What number added to 2, 20, 9, 34, will make the results proportional?

4. Find two numbers such that their sum, difference, and the sum of their squares are to each other as 4, 1, 17.

5. If $a : b :: c : d$ show that

$$a : b :: \sqrt{3a^2 + 5c^2} : \sqrt{3b^2 + 5d^2}.$$

6. Prove that a proportion taken by division is a true proportion.

7. The area of a circle varies as the square of its radius and the area of a circle is 154 sq. ft. when the radius is 7 ft. Find the area of the circle whose radius is 10 ft. 6 in.

8. The offing at sea varies as the square root of the height of the eye above sea-level, and the distance is 3 miles when the height is 6 ft. Find the distance when the height is 50 yds.

## SECTION XXIV.

## PROGRESSIONS.

1. Given $a$, $d$, and $n$ in an arithmetical progression, to find $l$.

2. Given $a$, $l$, and $r$ in a geometrical progression, to find the sum $s$.

   [NOTE. — The above problems are given frequently.]

3. The first and ninth terms of an arithmetical progression are 5 and 22. Find the sum of 21 terms.

4. Find the $n$th term of the series 2, $2\frac{1}{3}$, $2\frac{2}{3}$.

5. Find $d$ and $l$ when $a = 3$, $n = 15$, $s = -165$.

6. Insert 3 arithmetical means between $-9$ and 18.

7. Find the twelfth term of $\sqrt{2}$, $-2$, $2\sqrt{2}$, $-4$, etc.

8. Sum the infinite series $\frac{1}{3} + \frac{1}{3^2} + \frac{1}{3^3} +$ etc.

9. The sum of 3 numbers in arithmetical progression is 12, and the sum of their squares is 50. Find the numbers.

10. If a clock is constructed so as to strike up to 24, how many strokes will it make in the revolution of the index?

## SECTION XXV.

## PROGRESSIONS.

1. Given $a$, $l$, and $n$, in an arithmetical progression, to find $s$.

2. Given $a$, $n$, and $r$, in a geometrical progression, to find $l$.

[NOTE. — The above problems are given frequently.]

3. In an arithmetical progression, $s = -\frac{55}{4}$, $n = 20$, $a = \frac{1}{2}$. Find $d$.

4. Find the $(2n)$th term of 1, 3, 5, 7, etc.

5. Find $a$ and $n$ when $l = -47$, $d = -1$, $s = -1118$.

6. Insert 3 geometrical means between $\frac{1}{2}$ and 128.

7. Find the seventh term of $-\frac{1}{3}$, $\frac{1}{2}$, $-\frac{3}{4}$, etc.

8. Sum the infinite series $\frac{1}{5} + \frac{1}{25} + \frac{1}{125}$ .....

9. A traveller has a journey of 132 miles. He goes 27 miles the first day, 24 the second, and so on, travelling 3 miles less each day. In how many days will he complete his journey?

10. Find the sum of all the numbers which are less than 500 and are divisible by 11 without a remainder. [*Yale.*]

## SECTION XXVI.

### BINOMIAL THEOREM. PERMUTATIONS AND COMBINATIONS.

1. Expand by the binomial theorem $(\frac{1}{2}a^2 - \frac{2}{3}b)^5$.

2. Expand to 4 terms $\dfrac{1}{\sqrt{1+x^2}}$. [*Yale.*]

3. Expand $(2a - 3b)^{-2}$

4. Obtain 4 terms $\left(a^{\frac{3}{2}} + \dfrac{b^2}{2a^{\frac{1}{2}}}\right)^{20}$

5. Find the fifth term of $(x^{-1} - 2y^{\frac{1}{2}})^{11}$.

6. Second term of $\left(\sqrt[5]{a^3} - \dfrac{2}{17a^9}\right)^{51}$ [*Harvard.*]

7. Find the term independent of $x$ in $\left(3x - \dfrac{1}{x}\right)^4$.

8. Expand $(1 + 2x^2)^n$ to 4 terms.

9. Expand $(a^2 + 1 + a^{-2})^3$.

10. How many different amounts can be made up from 5 different coins?

11. In how many ways can 7 children form a ring?

12. I have 5 single volumes and a set of 3 volumes. In how many ways can I arrange these 8 books on a shelf, keeping the set together and in order?

## SECTION XXVII.

## BINOMIAL THEOREM. PERMUTATIONS AND COMBINATIONS.

1. Expand by the binomial theorem $\left(a - \frac{2b^2}{a}\right)^4$.

2. Expand to 4 terms $(a + x)^{-\frac{1}{4}}$.

3. Expand $(\sqrt{3} - 3\sqrt[3]{a})^4$. [*Technology.*]

4. Obtain the first 3 and last 3 terms of $(x - y)^{100}$.

5. Find the fourth term of $(2x - 3y)^{-5}$.

6. Fourth term of $\left(\frac{b}{\sqrt[3]{a^2}} - \frac{5}{1}a^{\frac{5}{3}}b^{-2}\right)^{27}$. [*Harvard.*]

7. Find the terms without radicals $\left(2\sqrt{a} - \sqrt{\frac{b}{a}}\right)^4$.

8. Expand to 5 terms $(1 + a)^{xy}$.

9. Expand $(e^x - e^{-x})^4$.

10. How many different signals can be made with 12 different flags by hoisting 4 at a time above each other?

11. How many combinations can be made from the word *Payson* taken 3 letters at a time?

12. At a whist party there are 6 ladies and 6 gentlemen. The host is to play with the most honored guest, and the hostess with the poorest player. In how many ways can the players be arranged if each man has a lady partner?

## SECTION XXVIII.

## UNDETERMINED COEFFICIENTS, LIMITS, LOGARITHMS.

1. Expand $\frac{2+x}{1+x-x^2}$ into a series of four terms by undetermined coefficients.

2. Develop $\frac{1+2x}{3+4x}$ into a series.

3. Separate $\frac{x^2}{(x^2-1)(x-2)}$ into partial fractions.

4. Separate $\frac{3x^2}{(x-1)^2(x+1)^2}$ into partial fractions.

5. Find the limit, when $x$ increases without limit, of

$$\frac{(x+1)(x^2-3)}{x^4-3x}.$$

6. Prove $\log_b m = \frac{\log_a m}{\log_a b}$.

7. Simplify $\frac{2.372 \times 7232 \times .003722}{(-22.37)(72230000)}$.

8. Find the value of $x$ in the equation $5^{\frac{2}{x}} = 30$.

## SECTION XXIX.

## UNDETERMINED COEFFICIENTS, LIMITS, LOGARITHMS.

1. Expand into a series by undetermined coefficients

$$\frac{1+2x}{1-x-x^2}.$$

2. Develop to four terms $\frac{3+x}{2-x-x^2}$.

3. Separate $\frac{1}{x^4-a^4}$ into partial fractions.

4. Separate $\frac{3x^2+x-2}{(x-2)^2(1-2x)}$ into partial fractions.

5. Evaluate $\frac{(x^2-1)(x^2+2)}{x^4-3}$ for $x \doteq \infty$.

6. Prove that $\log_b a \times \log_a b = 1$.

7. Simplify $\sqrt[3]{126}\sqrt{108} \div \sqrt[6]{1008}\sqrt[3]{162}$.

8. Given the amount of a given principal for a given number of years, to find the rate per cent.

## SECTION XXX.

## SPECIMEN PAPER.

1. Evaluation: The score of the Amherst-Technology game on Saturday was

$$\frac{e^2+f^2+g^2}{\sqrt{h}} \text{ to } g+\{f-[h-(e+g)]\}+\sqrt{g}.$$

Find numerical value if $e=f=g=h=4$.

2. Expression: A man's monthly salary is $\$x$. His weekly expenses are $\$w$, besides annual tax of $\$t$, and semi-annual insurance of $\$s$. How much does he save yearly?

3. Parentheses: $c-[2a-b-(3a-2b-\overline{4a-3b})]$. [*Sheffield.*]

4. Multiplication: $(\frac{1}{2}a^m+\frac{1}{3}a^{m-1}+\frac{1}{4})(\frac{1}{2}a-\frac{1}{3})$.

5. Division: $(a^2-2b^2-6c^2+ab-ac+7bc)$ by $(a-b+2c)$. [*Wor. Tech.*]

6. Formulas: Square $[(b-2)x+(1-b)]$.

7. Inspection: Divide $\left(a^4-\frac{1}{a^4}\right)$ by $\left(a-\frac{1}{a}\right)$ [*Mines.*]

8. Factoring: $x^2-y^2-z^2+2yz$. [*Yale.*]

9. G. C. Measure: $\begin{cases}12x^2-29x+14,\\18x^2+3x-10.\end{cases}$ [*Harvard.*]

10. Miscellaneous: H. C. F. by factoring or division

$$\begin{cases}x^4-(a^2+b^2)x^2+a^2b^2,\\x^4-(a+b)^2x^2+2ab(a+b)x-a^2b^2.\end{cases}$$ [*French Collection.*]

## SECTION XXXI.

## ADVANCED PROBLEMS.

1. A certain librarian spends every year a fixed sum for books. In 1886, the cost of his purchases averaged two dollars per volume; in 1887, he bought 300 more volumes than in 1886; and in 1888, 300 more volumes than in 1887. The average cost per volume was thirty cents lower in 1888 than in 1887. Find the number of volumes bought each year, and the fixed price paid for them. (Obtain two solutions.) [*Harvard.*]

2. A and B start at the same time from two towns and travel towards each other. When they meet B has travelled $a$ miles more than A; it will take A $b$ days longer to reach the town B left, and B $c$ days longer to reach the town A left. Find the distance between the towns. [*Harvard.*]

3. Three students A, B, and C, agree to work out a series of difficult problems in preparation for an examination; and each student determines to solve a fixed number every day. A solves 9 problems per day, and finishes the series 4 days before B; B solves 2 more problems per day than C, and finishes the series 6 days before C. Find the number of problems and the number of days given to them by each student. [*Harvard.*]

## SECTION XXXII.

### HARVARD.

[Write legibly and without crowding; give the work clearly, and find all possible answers. The shortest methods of work are preferred. Abridged processes of work may be used, but should be distinctly indicated.]

1. Solve the equation

$$\frac{2}{\frac{1}{b}\left(\frac{x}{2a}-2\right)}-\frac{b}{a}\left[\frac{4a}{\frac{4b}{x}-\frac{b}{a}}-\frac{1}{\frac{1}{x}-\frac{1}{b}}\right]=0.$$

2. Two tanks A and B are discharging water; A at the rate of $x$ barrels per hour, and B at the rate of $(x+100)$ barrels per hour. At $(1+y)$ hours after noon, A contains 470 barrels less than at noon; and at a time $(1-y)$ hours after noon, B contains 400 barrels less than at noon. Find the rate at which each tank is discharging water, and the times $(1+y)$ and $(1-y)$ hours after noon. Obtain two sets of answers, and interpret negative results.

3. Find the Greatest Common Measure and the Least Common Multiple of $12x^2-29x+14$ and $18x^2+3x-10$.

4. Find the fourth term of $\left(\sqrt{a}-\frac{b^7}{6a^5}\right)^{81}$

5. Find a mean proportional between

$$4x^3-3x-1 \quad \text{and} \quad x-1.$$

## SECTION XXXIII.

### YALE.

EXAMINATION FOR ADMISSION.

1. Express $\dfrac{2\sqrt{3}-3}{1+\sqrt{3}-\sqrt{5}}$ in an equivalent fraction having a rational denominator.

2. Divide $81x-16$ by $3x^{\frac{1}{4}}+2$.

3. Find the cube of $1-\dfrac{1-x^{\frac{1}{2}}}{1+x^{\frac{1}{3}}}$.

4. Find the square root of $27-12\sqrt{5}$.

5. Solve the equation $1-7x^2=2ax-bx^2$.

6. Solve the equations $\begin{cases} xy=a, \\ x^2+y^2=b. \end{cases}$

7. Expand $\dfrac{4}{(a-2x)^7}$ into a series, giving five terms.

## SECTION XXXIV.

### VASSAR.

1. Prove that the product of two quantities is equal to the product of their G. C. D. and L. C. M.

   Find G. C. D. and L. C. M. of

   $$8x^3+27;\quad 16x^4+36x^2+81;\quad 6x^2+5x-6.$$

2. Simplify $\sqrt{\frac{a+b}{a-b}}\sqrt{\frac{a+b}{a-b}}\times\sqrt[3]{\frac{a-b}{a+b}}\sqrt{\frac{a-b}{a+b}}$.

3. Solve the equations: (1) $\dfrac{a-\sqrt{a^2-x^2}}{a+\sqrt{a^2-x^2}}=b.$

   (2) $x^3+y^3=56,$
   $x+y=2.$

4. Define *root* of an equation. How many roots has a simple equation? Form the equation whose roots are $a$ and $-b$.

5. Define $a^m$, $m$ being a positive integer, and prove $(a^m)^n=a^{mn}$.

   Multiply together $\sqrt[3]{abc}$, $a^{\frac{4}{3}}b^{-1}c^{\frac{1}{3}}$, $a^{-\frac{2}{3}}b^{\frac{2}{3}}c^{-1}$.

   Write down the values of $3^4$; $5^0$; $0^2$; $16^{-\frac{1}{4}}$; $2^{-\infty}$; $4^{\frac{1}{2}}$.

6. There are three numbers in arithmetical progression whose sum is 15. If 1, 3, 9 are added to them respectively, they are then in geometrical progression. Find the numbers.

7. Explain the meaning of "*mean proportional*," "*fourth proportional*."

   If $a:b::c:d$, prove that each of their ratios is equal to $(a+mc):(b+md)$.

## SECTION XXXV.

### WELLESLEY.

1. Factor $x - 3a\sqrt{x} - 28a^2$; $m^4 + m^2n^2 + n^4$ and $a^{\frac{1}{2}}b^{2m}c^{-z} + a^{-\frac{1}{2}}b^{-2m}c^2 - 2$.

2. Simplify $\left[\dfrac{a^{\frac{x}{x-y}}\sqrt[y]{b^{-(y-x)}}}{a^{\frac{y}{x-y}}\sqrt[y]{a^{x+y}b^x}}\right]^{-y}$.

3. Add $2\sqrt{125} - \sqrt[4]{\frac{25}{16}} + \sqrt[3]{81} - (-512)^{\frac{1}{3}} + \sqrt[3]{192}$ and $[7\sqrt[6]{9} + 2\sqrt{120} + 3(\frac{80}{9})^{-\frac{1}{2}} + 3\sqrt{\frac{1}{5}} + 7(\frac{9}{5})^{\frac{1}{2}}](\sqrt{-1})^6$.

4. Define simultaneous equations.

5. Sum the series $1, \frac{1}{3}, -\frac{1}{3}$ ..... to 9 terms, and derive the formula for the sum of a series.

6. Solve for $x$ in the following equations.

   1. $\dfrac{2}{x+\sqrt{2-x^2}} + \dfrac{2}{x-\sqrt{2-x^2}} = x.$

   2. $3x^n\sqrt[3]{x^n} + 2\dfrac{x^n}{\sqrt{x^n}} = 16.$

7. Find $x$ and $y$ from the equations

$$\left.\begin{aligned}(x-y)^3 : x^3 - y^3 &= 3 : 7\\ xy &= 4\end{aligned}\right\}$$

   using the principles of proportion as far as possible. Prove the most important of the principles of proportion used.

8. A distributes \$180 in equal sums among a certain number of people. B distributes the same sum, but gives to each person \$6 more than A, and gives to 40 persons less than A does. How much does A give to each person?

## SECTION XXXVI.

### CORNELL.

1. Define the H. C. F. and L. C. M. of two polynomials, and explain fully the theory of the method of finding them.

2. Determine the common factors of
$12x^3 - 8x^2 - 3x + 2$ and $16x^3 + 12x^2 - 4x - 3$,
and hence find 3 values of $x$, which when substituted in the expression
$[(12x^3 - 8x^2 - 3x + 2)] + [(16x^3 + 12x^2 - 4x - 3)]$
will give zero for the result.

3. If the roots of the quadratic $x^2 - px + q = 0$ are two consecutive integers, prove $p^2 - 4q - 1 = 0$.

4. Extract the square root of $47 - 12\sqrt{15}$, and find the value, when $x = \sqrt{3}$, of $\dfrac{2x-1}{(x-1)^2} - \dfrac{2x+1}{(x+1)^2}$.

5. For what values of $m$ will the equation
$x^2 - 2(1 + 3m)x + 7(3 + 2m) = 0$ have equal roots.

6. Solve the equations:

(a) $\dfrac{x - \frac{1}{2}}{x-1} - \dfrac{3}{5}\left(\dfrac{1}{x-1} - \dfrac{1}{3}\right) = \dfrac{23}{10(x-1)}$.

(b) $\begin{cases} ax - by = 2ab, \\ 2bx - 2ay = 3b^2 - a^2. \end{cases}$ (c) $\begin{cases} y^2 + xy = 4, \\ x^2 + 2y^2 - xy = 8. \end{cases}$

7. Simplify $\sqrt[mn]{\dfrac{a^m}{a^n}} \times \sqrt[np]{\dfrac{a^n}{a^p}} \times \sqrt[pm]{\dfrac{a^p}{a^m}}$.

8. A line 6 in. long is produced. Find the length of produced part so that the rectangle contained by half the line and the line made up of the half and the part produced may be equal to the square on the produced part.

## SECTION XXXVII.

### BRYN MAWR.

[Students are expected to answer questions in each of the divisions B and C of the paper. Division A is in Arithmetic.]

**B.**

**5.** Prove the identity:

$$(a^2+b^2+c^2)(d^2+e^2+f^2)-(ab+be+cf)^2=(bf-ce)^2 + (cd-af)^2+(ae-bd)^2.$$

**6.** Solve the equations for all values of $x$:

$$(1)\ \begin{cases} ax-cy+cz=a \\ bx+cy\ \ \ =b \\ cx-cz\ \ \ =c \end{cases} \text{ also } (2)\ \begin{cases} 35x^2+49xy=x \\ x^2=4y^2 \end{cases}$$

**7.** Prove that $A=\dfrac{-b+\sqrt{b^2-4ac}}{2a}$ is a root of

$ax^2+bx+c=0.$

Show that if B is the other root, $AB$ must $=\dfrac{c}{a}$.

**C.**

**8.** Explain as clearly as you can why $a^{-1}$, $a^0$, $a^{-\frac{3}{4}}$, are said to be equal to $\dfrac{1}{a}$, 1, $\dfrac{1}{a^{\frac{3}{4}}}$.

**9.** A varies as the square of B and inversely as the square of C. Also $A=4$ when $B=1$, $C=2$.

What is the value of $\dfrac{B^2+A^2}{B^2-A^2}$ when $B=2$, $C=2$?

**10.** What is the $k$th term in the arithmetical progression whose first term is $a$ and $n$th term is $l$?

Sum the series $\dfrac{1}{3}+\dfrac{1}{3^2}+\dfrac{1}{3^3}+\cdots\cdots$ to infinity.

How many terms of the series $1+3+5+7+$ must be taken in order that the sum may be $(r-1)^2$ where $r$ is a positive integer?

## SECTION XXXVIII.

## TECHNOLOGY (BOSTON).

1. Reduce to their lowest terms

$$\frac{x^3-16x}{x^2-3x-4} \text{ and } \frac{a^2+2a(a+1)+4}{a^4-4}.$$

2. Show that

$$\left(\sqrt{\frac{x}{y}}-\sqrt{\frac{y}{x}}\right)^4=\left(\frac{x}{y}-\frac{y}{x}\right)^2-4\left(\sqrt{\frac{x}{y}}-\sqrt{\frac{y}{x}}\right)^2.$$

3. Show that

$$\left(\frac{-1+\sqrt{-3}}{2}\right)^3+\left(\frac{-1-\sqrt{-3}}{2}\right)^3=2.$$

4. Solve the simultaneous equations

$$x(y+1)=8;\ y(x+2)=12.$$

5. Find the square root of

$$x^6-4x^5+20x^2+16x+16.$$

6. Solve the equation

$$\sqrt[3]{28}-x+\sqrt[3]{x}=4.$$

7. Form the quadratic equation whose roots are $\frac{2}{3}$, $\frac{3}{2}$, and $\frac{2}{3}$.

8. The sum of three numbers in arithmetical progression is 18. If the first be increased by 1, and the third by 2, they will be in geometrical progression. What are the numbers?

## SECTION XXXIX.

## SHEFFIELD SCIENTIFIC SCHOOL.

### Entrance Examination.

### ALGEBRA FROM QUADRATICS.

[Note. — State at the head of your paper what text-book you have studied on the subject and to what extent.]

1. Solve the equation $ax^2 + bx + c = 0$, and point out what relation must exist between the coefficients in order that the roots may be equal.

2. Determine by inspection the roots of the equation
$$x(x+2)(x^2-4) = 0.$$

3. Solve the equation
$$x^2 - x + 5\sqrt{2x^2 - 5x + 6} = \frac{3x + 33}{2}.$$

4. There are 20 things of one kind and 10 of another; how many different sets can be made, each containing 3 of the first kind and 2 of the second?

5. Insert 3 arithmetical means between 4 and 20.

6. Expand $\frac{1-x}{1+x+x^2}$ into a series by the method of undetermined coefficients.

7. Find the limit of $\frac{(x+1)(x^2-3)}{x^4-3x}$ when $x$ is increased without limit.

## SECTION XL.

## PRINCETON.

[State what text-books you have read.]

1. (*a*) Find G. C. D. of

$$6x^2+13x-5 \text{ and } 3x^3+2x^2+2x-1.$$

(*b*) Factor $(nx-x+y-ny)$; also $(9a^2b^2-y^2)$.

2. Find the square root of

$$x^4+4xy^2+y^{-2}+4x^{\frac{5}{2}}y-2x^2y^{-1}-4x^{\frac{1}{2}}.$$

3. Solve the equations

$$\frac{a-x}{bc}+\frac{b-x}{ca}+\frac{c-x}{ab}=0.$$

4. Solve the equations

(*a*) $\dfrac{x}{4}-\dfrac{21-x}{4-x}=1,$

(*b*) $(x-1)^{-\frac{4}{3}}-5(x-1)^{-\frac{2}{3}}+1=0.$

5. Solve the simultaneous equations

$$\frac{7-2x}{5-3y}=\frac{3}{2};\quad y-x=4.$$

6. Solve the simultaneous equations

$$\frac{3x-2}{y+5}+\frac{y}{x}=2;\quad x-y=4.$$

7. What number must be added to $m$ and to $n$, in order that the sums may be in the ratio of $p$ to $q$?

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# MATHEMATICS.

## WENTWORTH'S SERIES.

THE United States Bureau of Education, in Circular of Information, No. 3, 1890, says, on the subject of mathematical text-books, "If we were called upon to name the writer whose books have met with more wide-spread circulation during the past decennium that those of any other author, we should answer, *Wentworth*. . . . In point of scientific rigor, Wentworth's books are superior to the popular works of preceding decades."

These two points indicate the two leading characteristics of these remarkable books. They combine mathematical scholarship with all the elements of school-room popularity. The author's aim was to be helpful. He has made a study of his subjects with the psychology and probable capacity of the students constantly in mind. His books enable the average student not merely to learn something, but to master the study, and at the same time they give the brighter ones a plenty to exercise their faculties upon.

There is no attempt to make a parade of learning. On the other hand, there is no absolution from solid work. Everything is made simple, practical, direct, and thorough. Mental energy is economized. The teacher has to use his strength only in the necessary work of teaching, and the pupil his only in the necessary work of acquiring.

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